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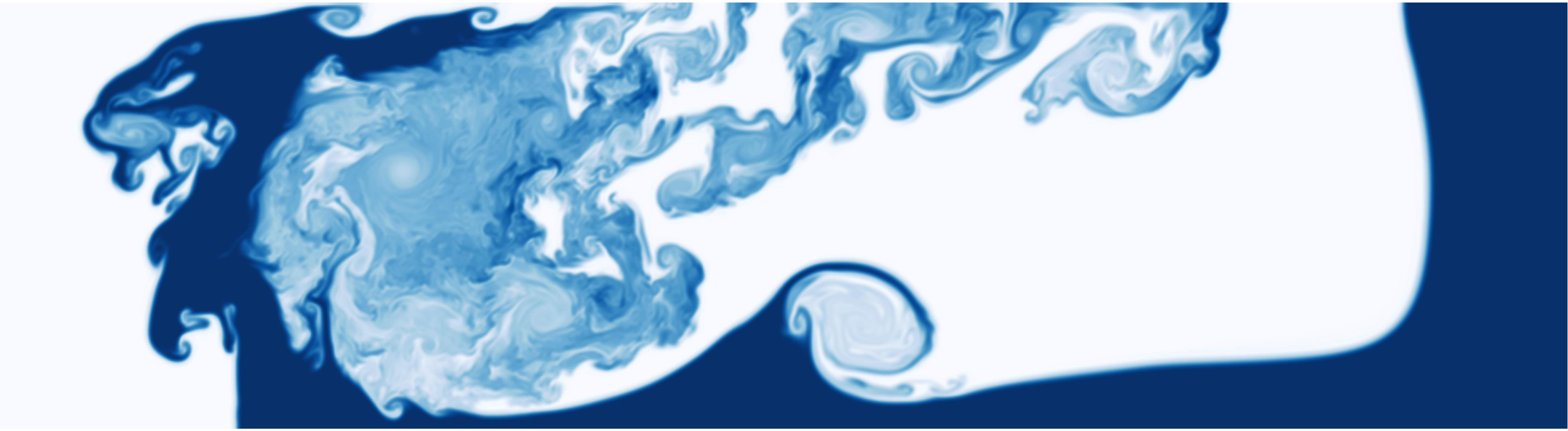

Shock Induced Turbulent Mixing

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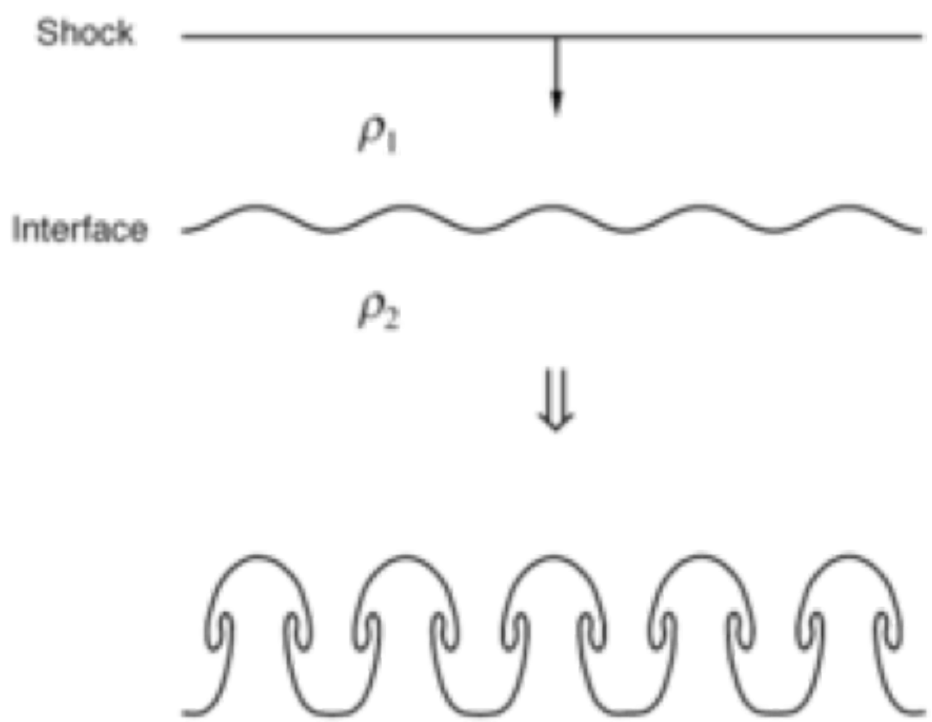
Outline

- Introduction - Richtmyer-Meshkov Instability
- Classical RM problem
- Inclined interface vs. single mode interface
- Numerical technique
- Problem setup
- Results
- Effect of 3D perturbations
- Conclusions

Richtmyer-Meshkov (RM) Instability

- Interaction of a material interface with a shockwave
- Predicted theoretically by Richtmyer (1960) and shown experimentally by Meshkov (1969)
- Similar to Rayleigh-Taylor in mechanism
- Baroclinic vorticity generation causes amplification of perturbations
- Linear models for small amplitude sinusoidal perturbations

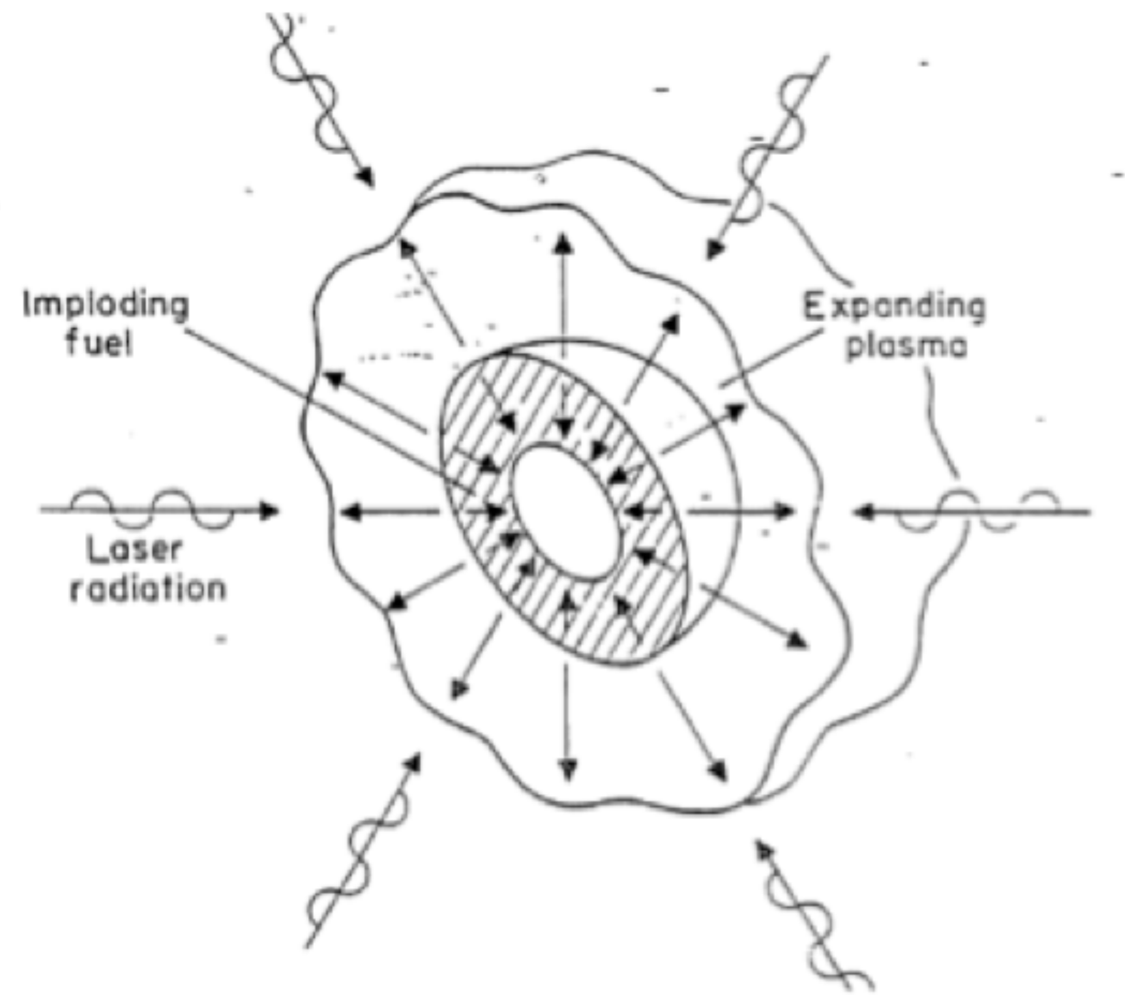
Classical RM configuration



$$\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \boldsymbol{u} + \nu \nabla^2 \boldsymbol{\omega} + \underbrace{\left(\frac{1}{\rho^2} \nabla \rho \times \nabla p \right)}_{\text{Baroclinic vorticity generation}}$$

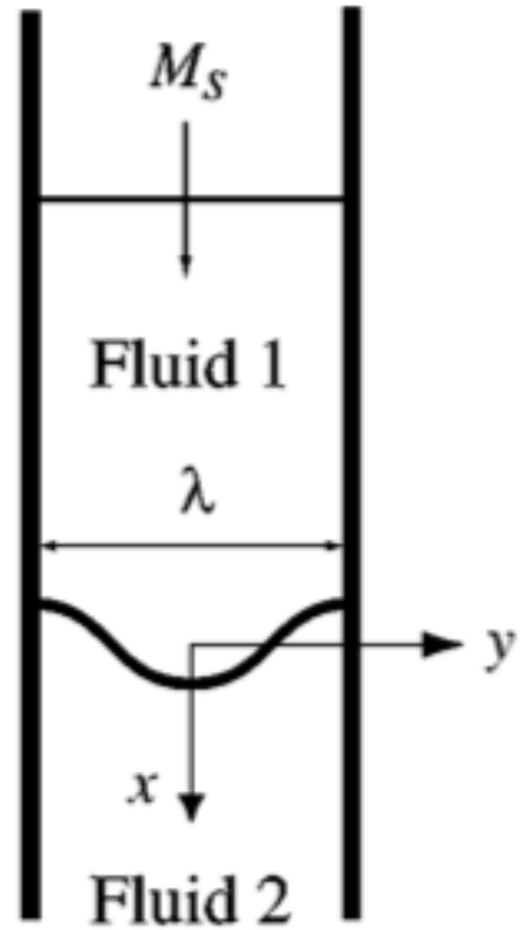
Applications

- Inertial Confinement Fusion (ICF)
    - Critical to achieve energy break-even
- Stellar evolution models to explain lack of stratification
- Mixing in supersonic and hypersonic air-breathing engines
- Aim is to develop predictive capabilities
- Simulations key to bridging gap between experiments, theory and modeling



The classical RM problem

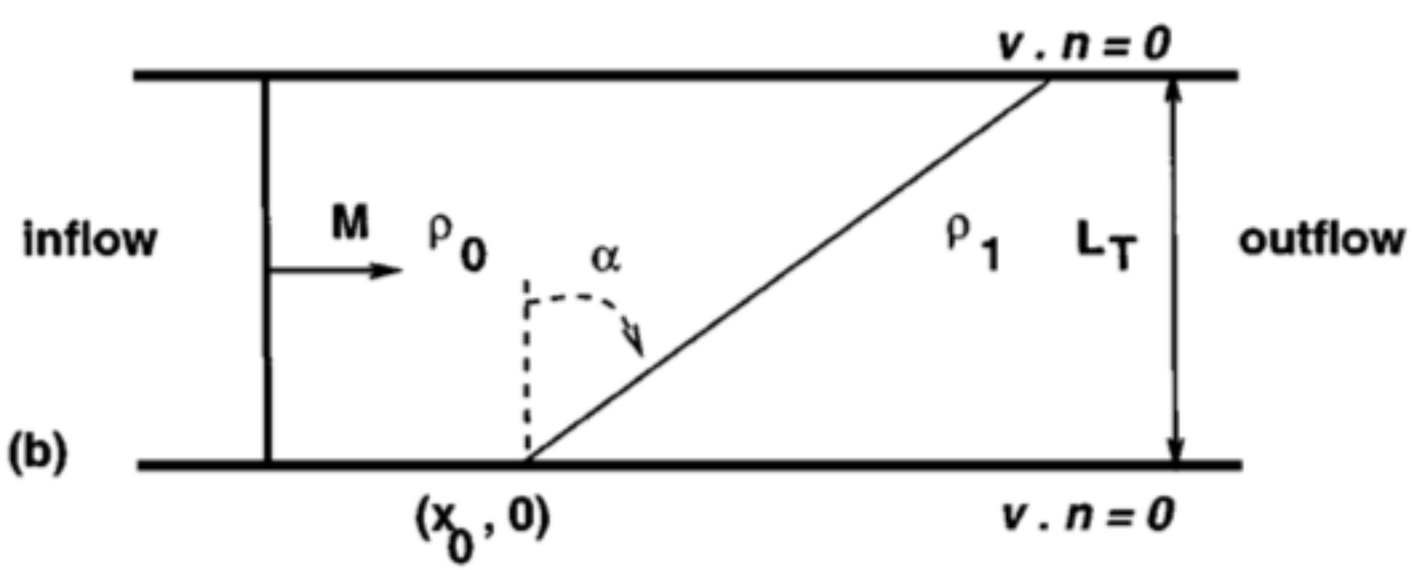
- First model by Richtmyer for small amplitude sinusoidal perturbations
- Many models that work well in the linear regime
- Some extensions to early non-linear times
- No net circulation deposition



From Brouillete (1990)

Inclined interface RM

- No existing model for interface evolution
- Intrinsically non-linear from early times for modest interface angles
- Almost constant vorticity deposition along the interface
- Easier to study experimentally



From Zabusky ('99)

Governing Equations

- We solve the compressible multi-species Navier Stokes equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial (\rho \boldsymbol{u})}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u} + p\underline{\boldsymbol{\delta}} - \underline{\boldsymbol{\tau}}) = 0$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + p)\boldsymbol{u}] - \nabla \cdot (\underline{\boldsymbol{\tau}} \cdot \boldsymbol{u} - \boldsymbol{q_c} - \boldsymbol{q_d}) = 0$$

$$\frac{\partial \rho Y_i}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} Y_i) - \nabla \cdot (\rho D_i \nabla Y_i) = 0$$

$$p(\rho e, Y_1, Y_2, ..., Y_K) = (\overline{\gamma} - 1)\, \rho e$$

Numerical technique

- Miranda code developed at LLNL (Cook '07)
- Compressible, multi-species solver
- 10^{th} order compact finite differencing (Lele '92) in space
- 4^{th} order Runge Kutta time integrator
- LAD scheme for generalized curvilinear coordinates (Kawai '08) for shock and interface capturing

$$\mu = \mu_f + \mu^*$$

$$\beta = \beta_f + \beta^*$$

$$\kappa = \kappa_f + \kappa^*$$

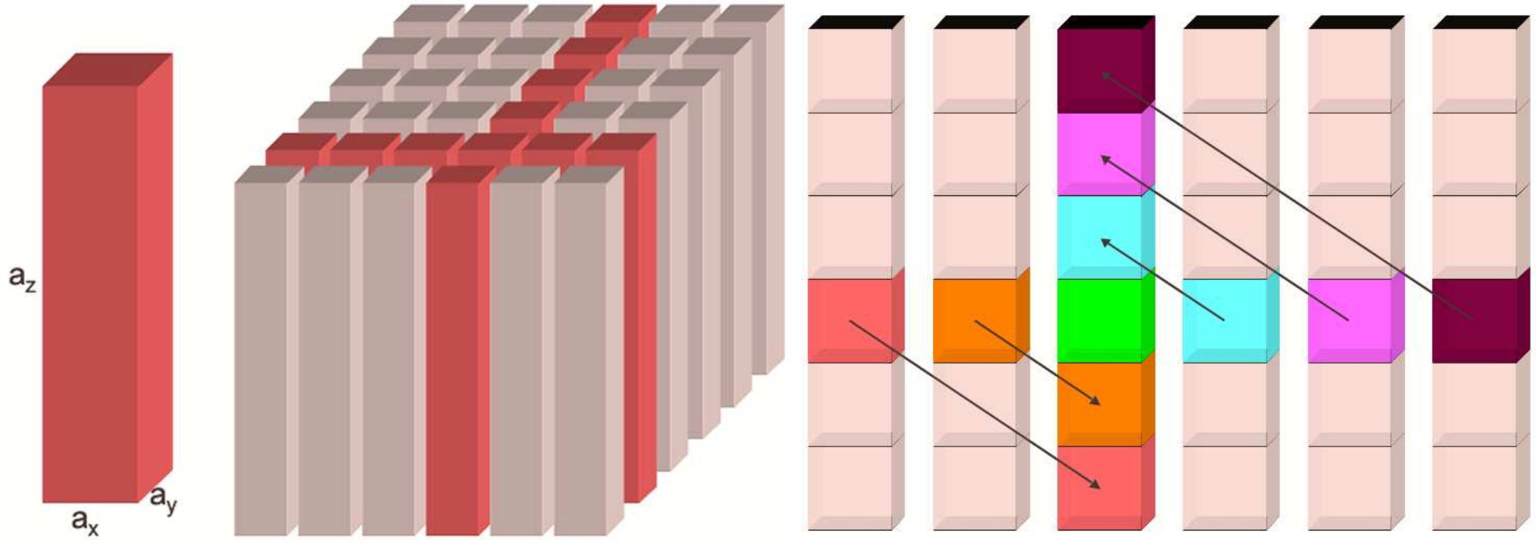
$$D_i = D_{f,i} + D_i^*$$

The Miranda Code

- 10th order Pade scheme for derivative computation

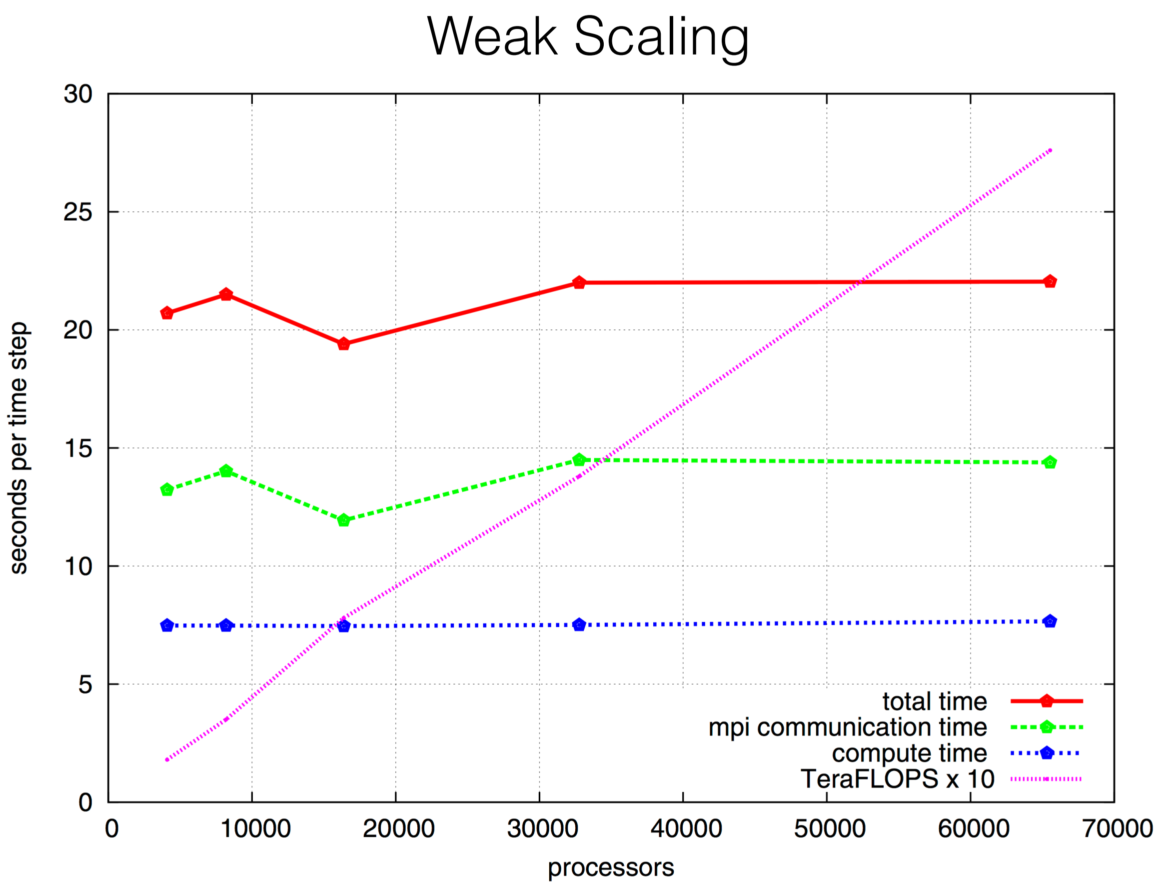
$$Af' = Bf$$

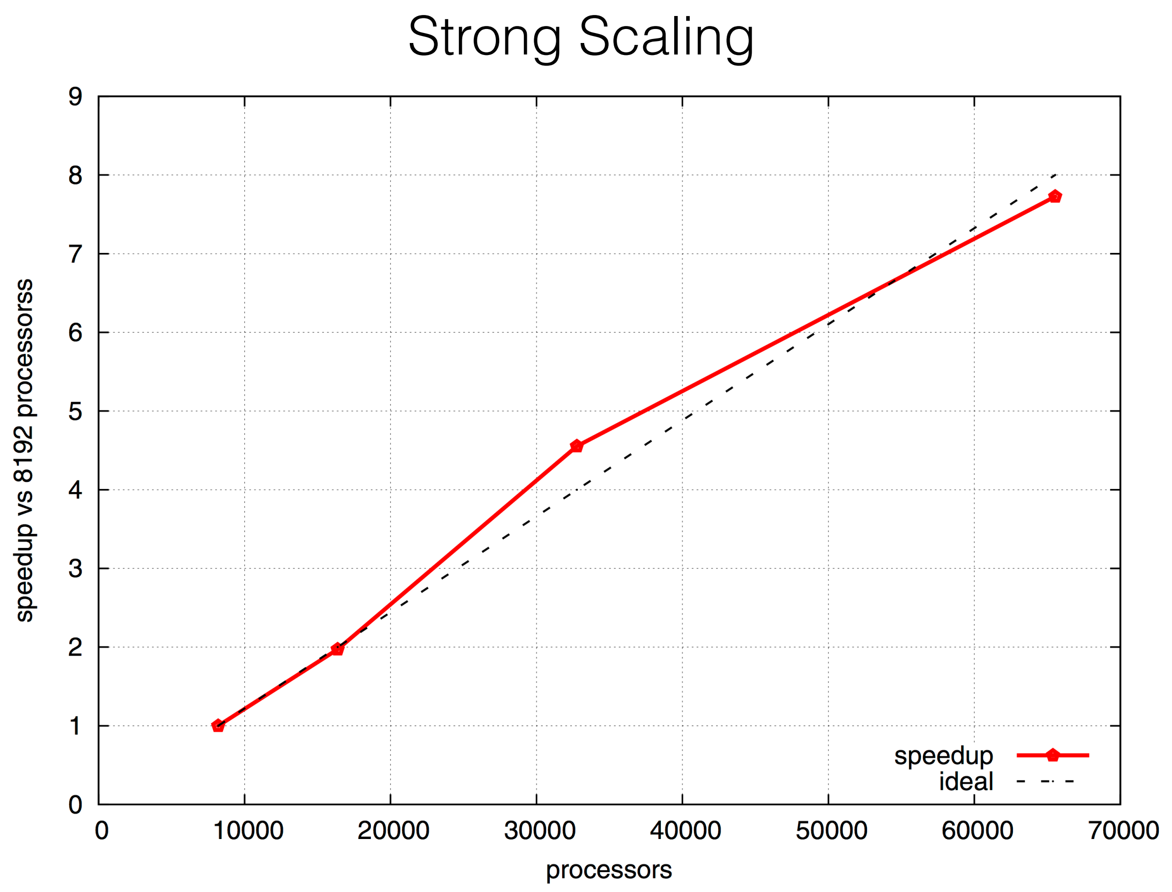
- Need to solve pentadiagonal system
- Two approaches
    - Direct block parallel pentadiagonal solves (BPP)
    - Transpose algorithm with serial pentadiagonal solves
- Transpose algorithm shown to scale very well up to 65,536 processors



From Cook et. al. (2005)

The Miranda Code

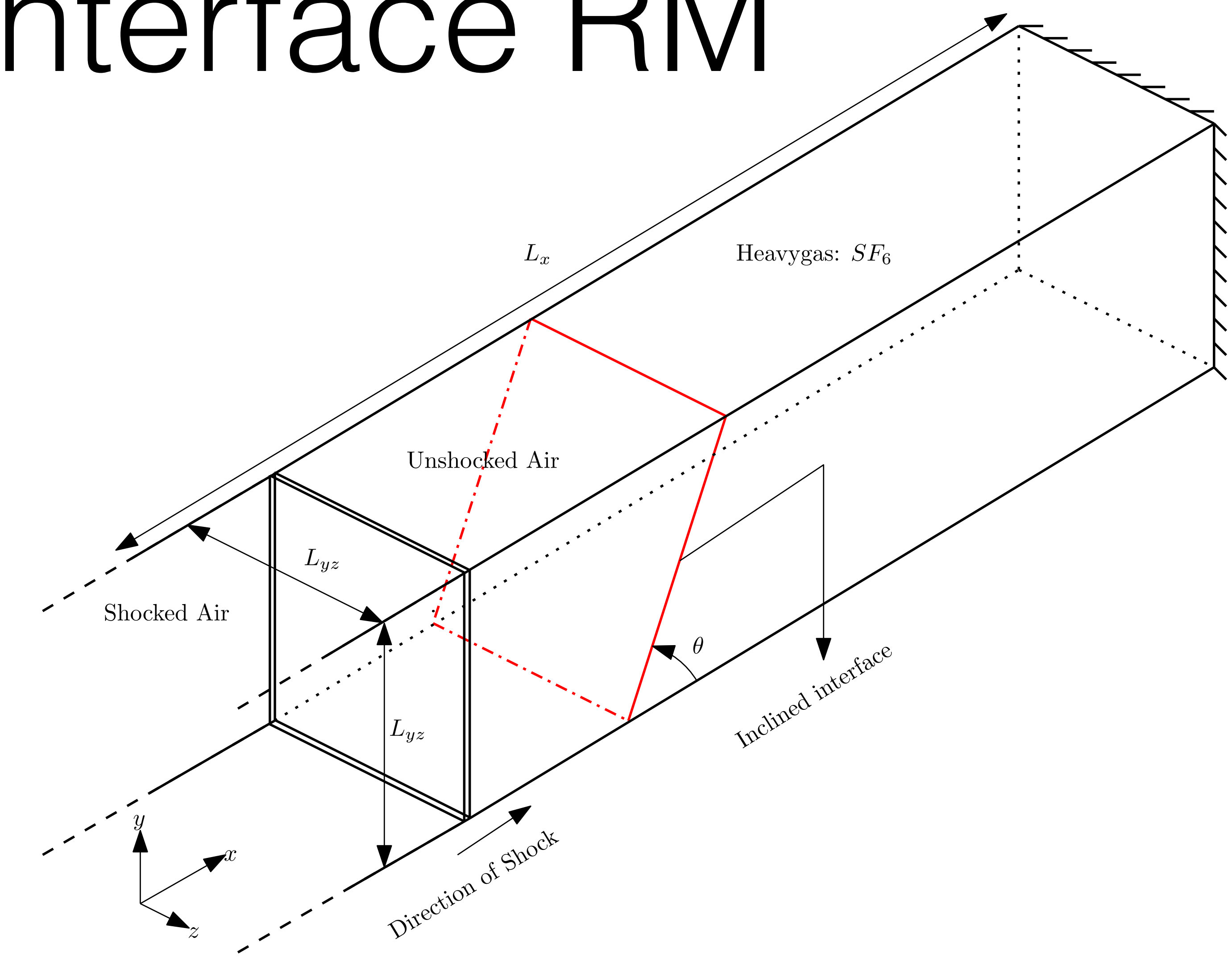




From Cook et. al. (2005)

Inclined interface RM

- No existing model for interface evolution
- Intrinsically non-linear from early times for modest interface angles
- Almost constant vorticity deposition along the interface
- Easier to study experimentally
- Based on experimental setup in the Inclined Shock Tube Facility at Texas A&M
- Slip walls in transverse (y) direction
- Isotropic 3D cartesian grid



$$L_{yz} = 11.4\ cm$$

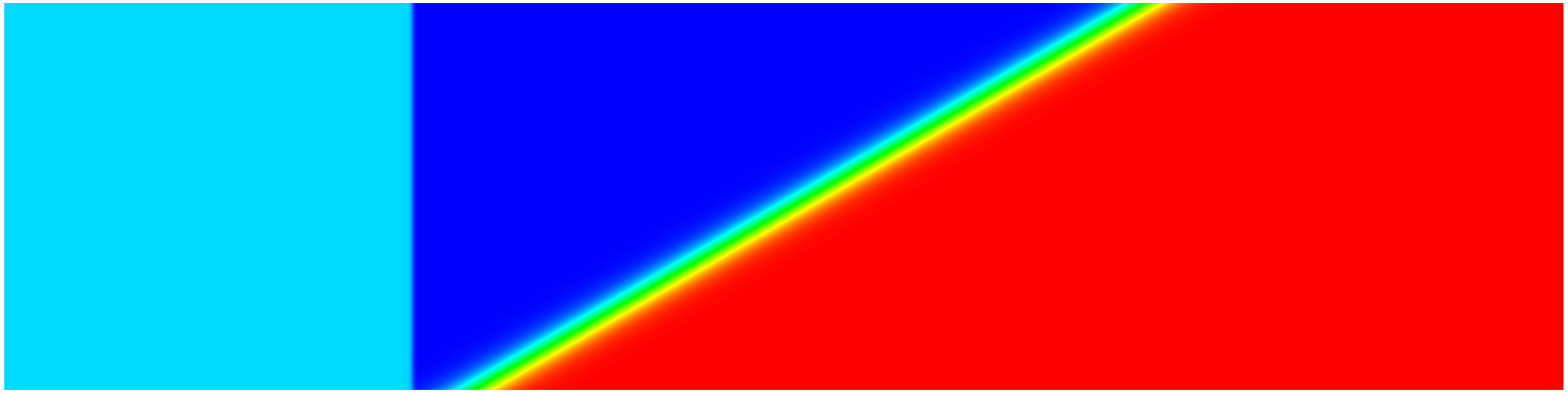
$$\theta = 30^\circ$$

$$M_{\text{shock}} = 1.5$$

$$A = \frac{\rho_{\text{SF}_6} - \rho_{\text{Air}}}{\rho_{\text{SF}_6} + \rho_{\text{Air}}} = 0.67$$

Time epochs

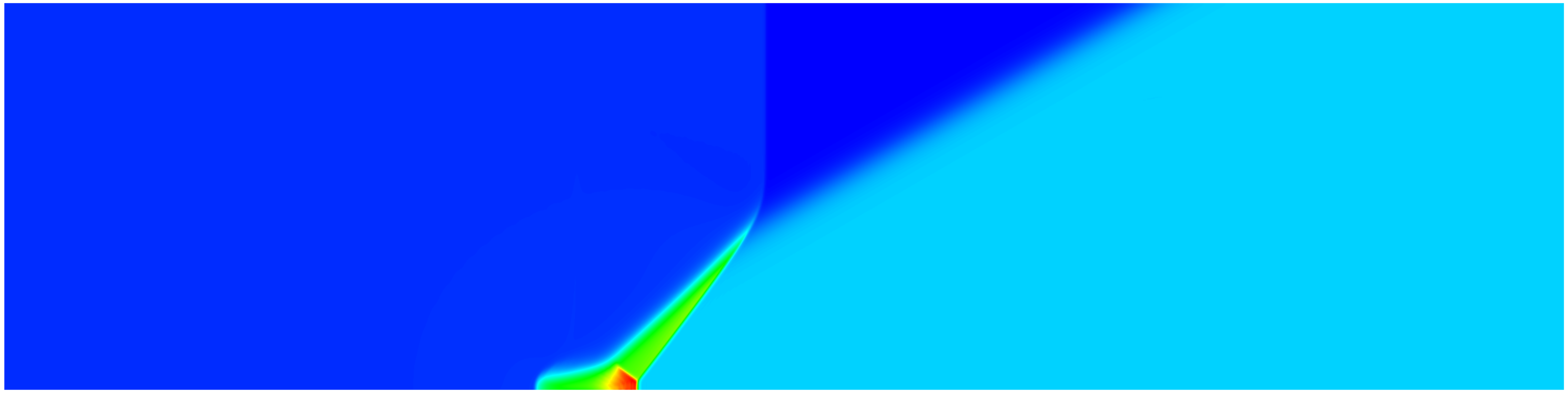
- Before interaction (initial condition, t = 0 ms)



Density field

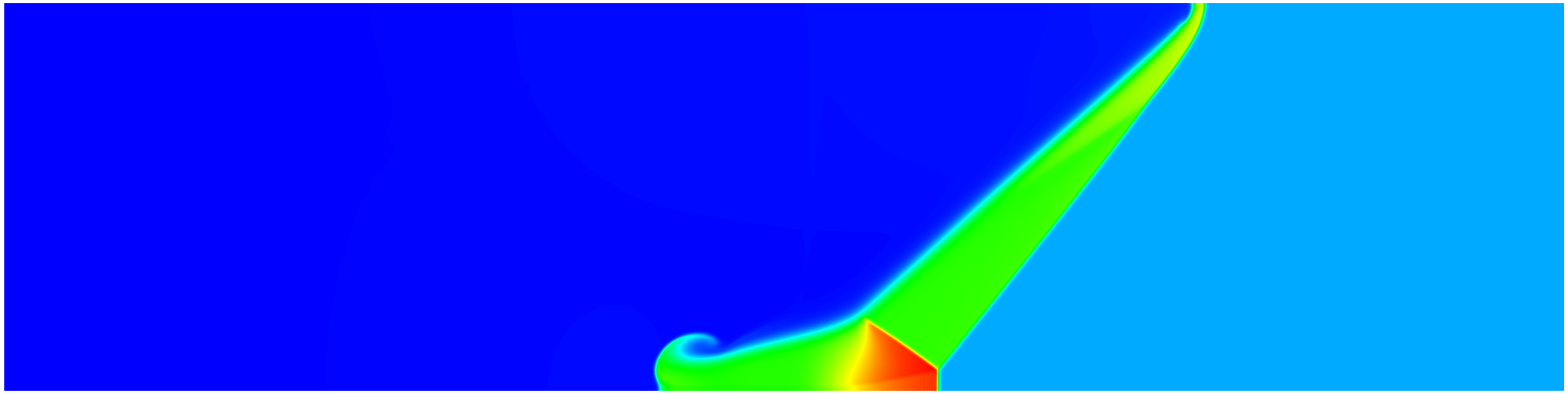
Time epochs

- First interaction of the shock and interface (t = 0.2 ms)



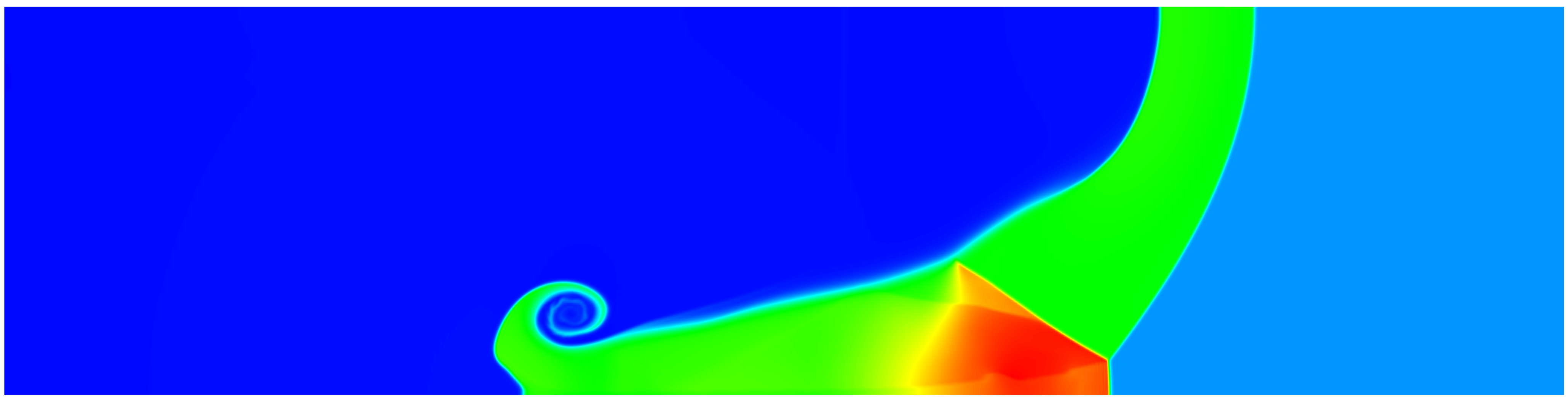
Time epochs

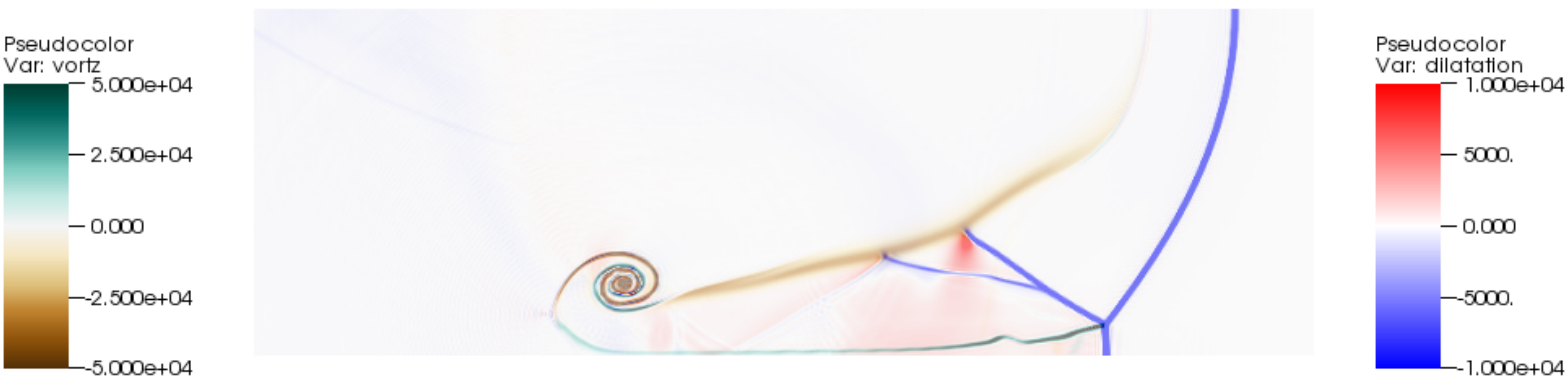
- Shock fully passes through the interface (t = 0.5 ms)



Time epochs

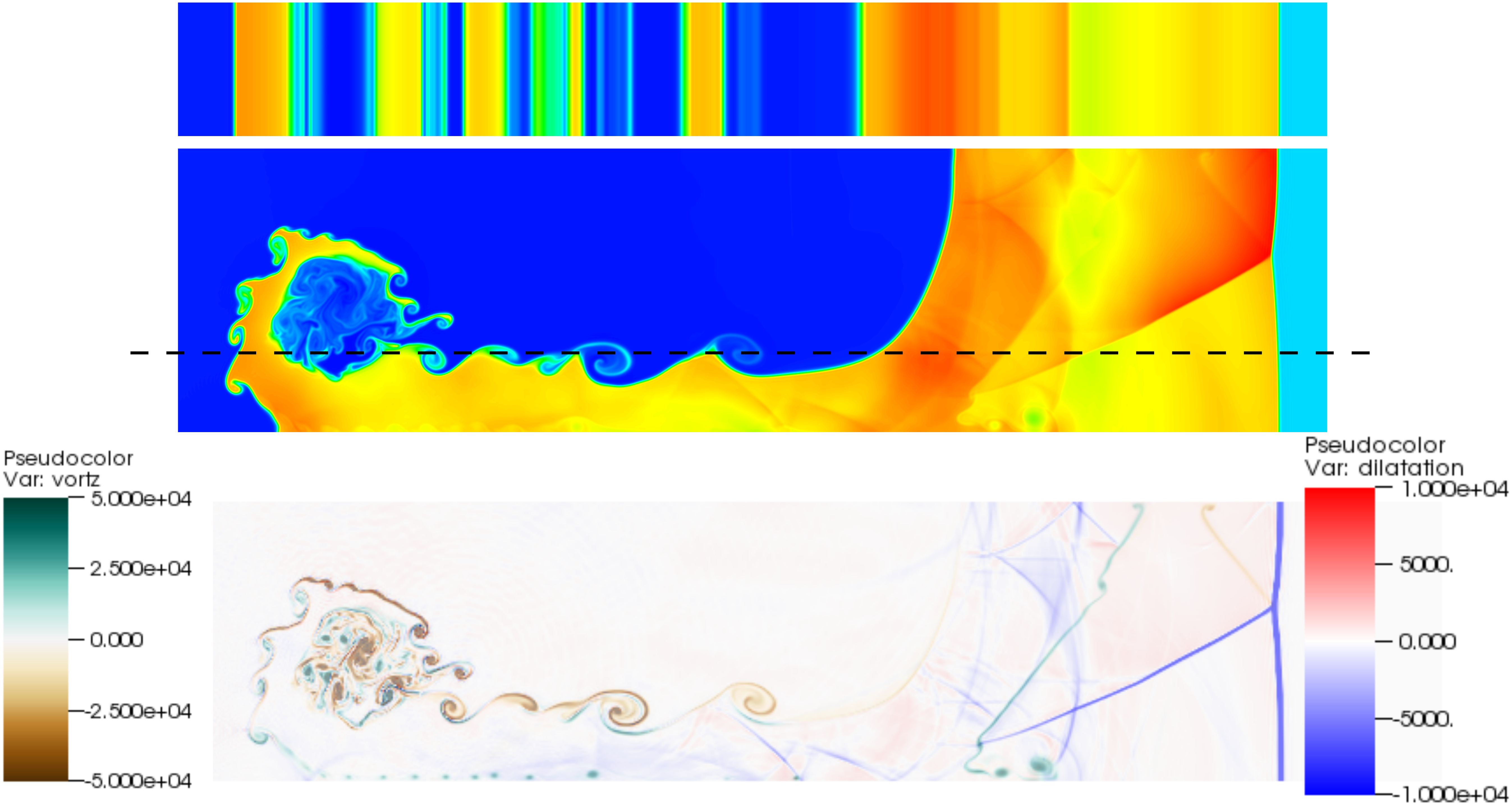
- Formation of a coherent wall vortex (t = 1.0 ms)





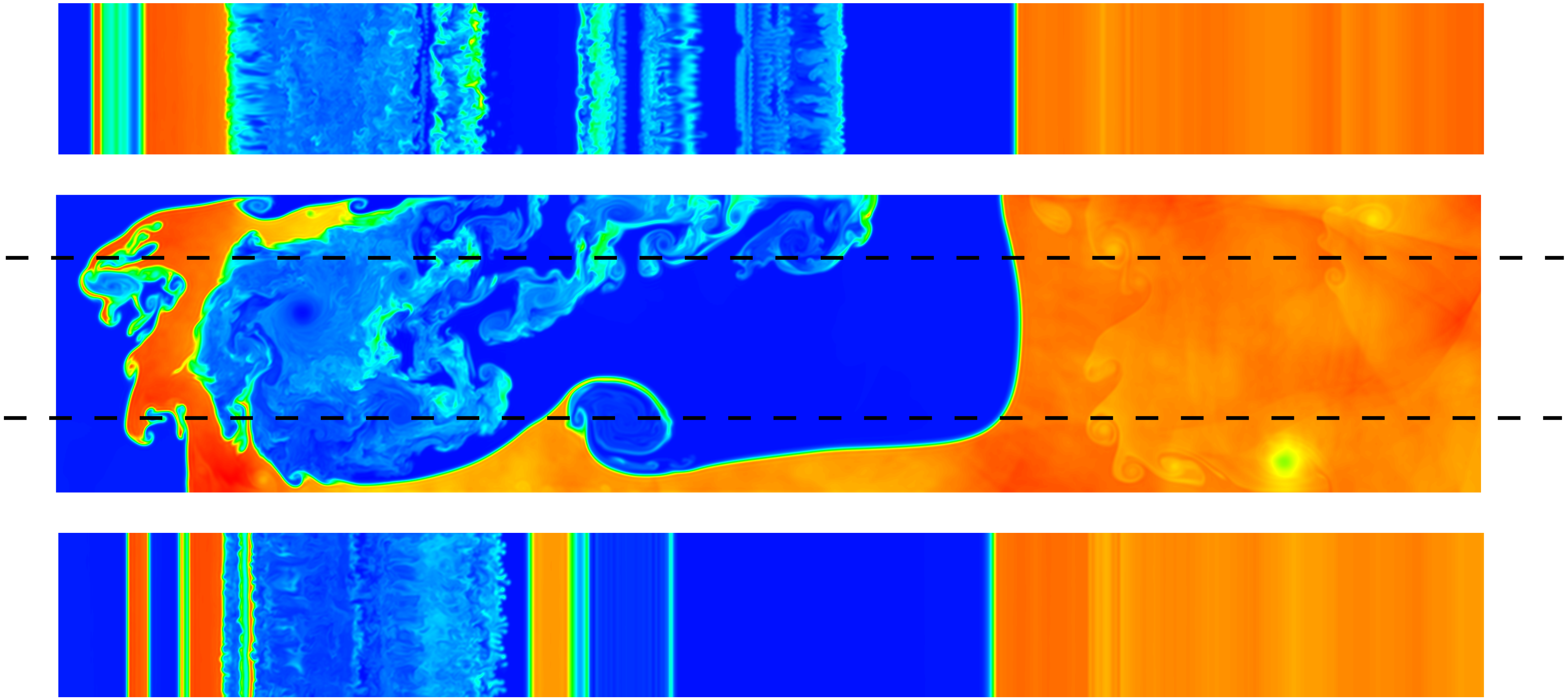
Time epochs

- Kelvin-Helmholtz rollers (t = 2.5 ms)

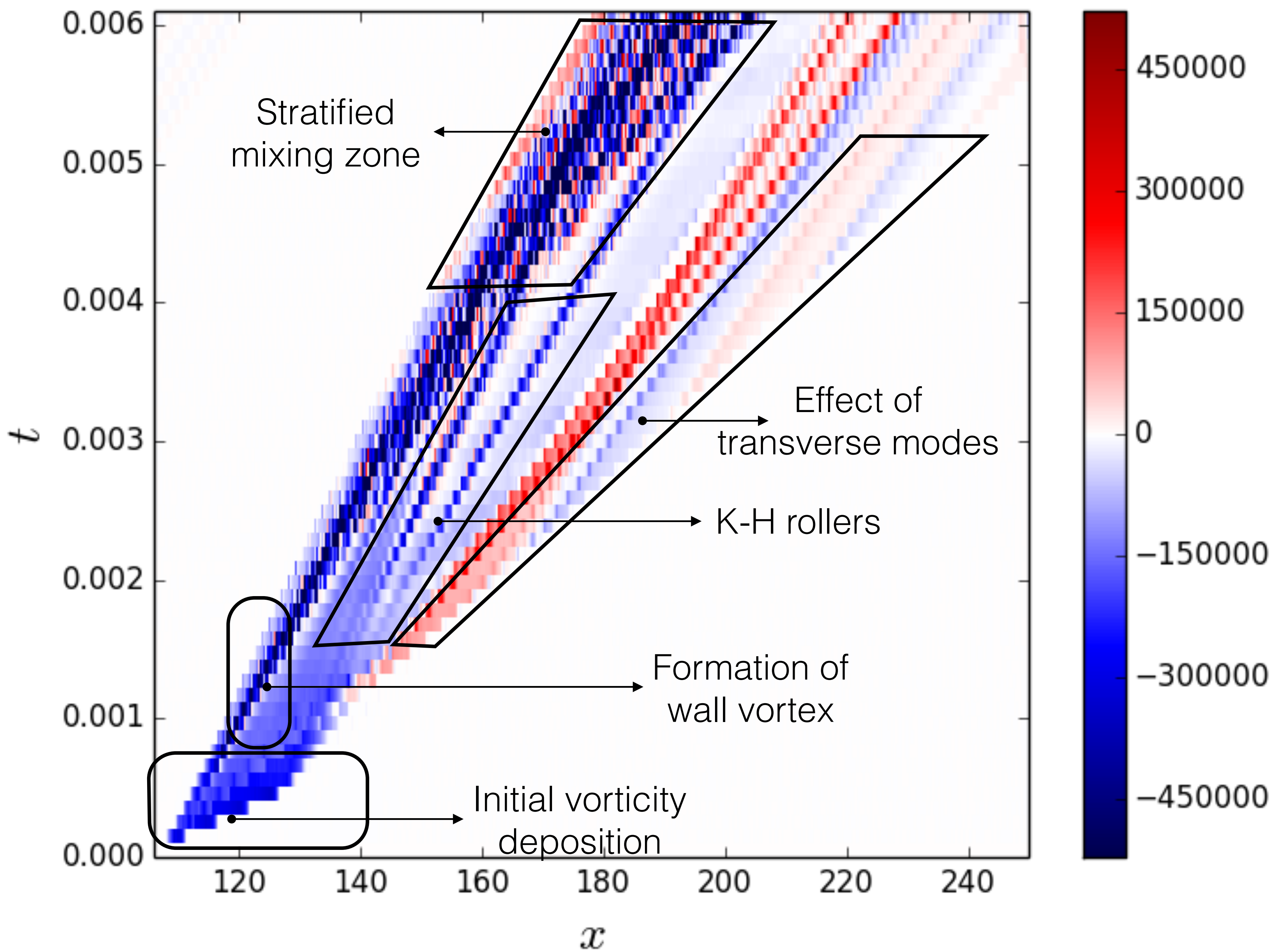


Time epochs

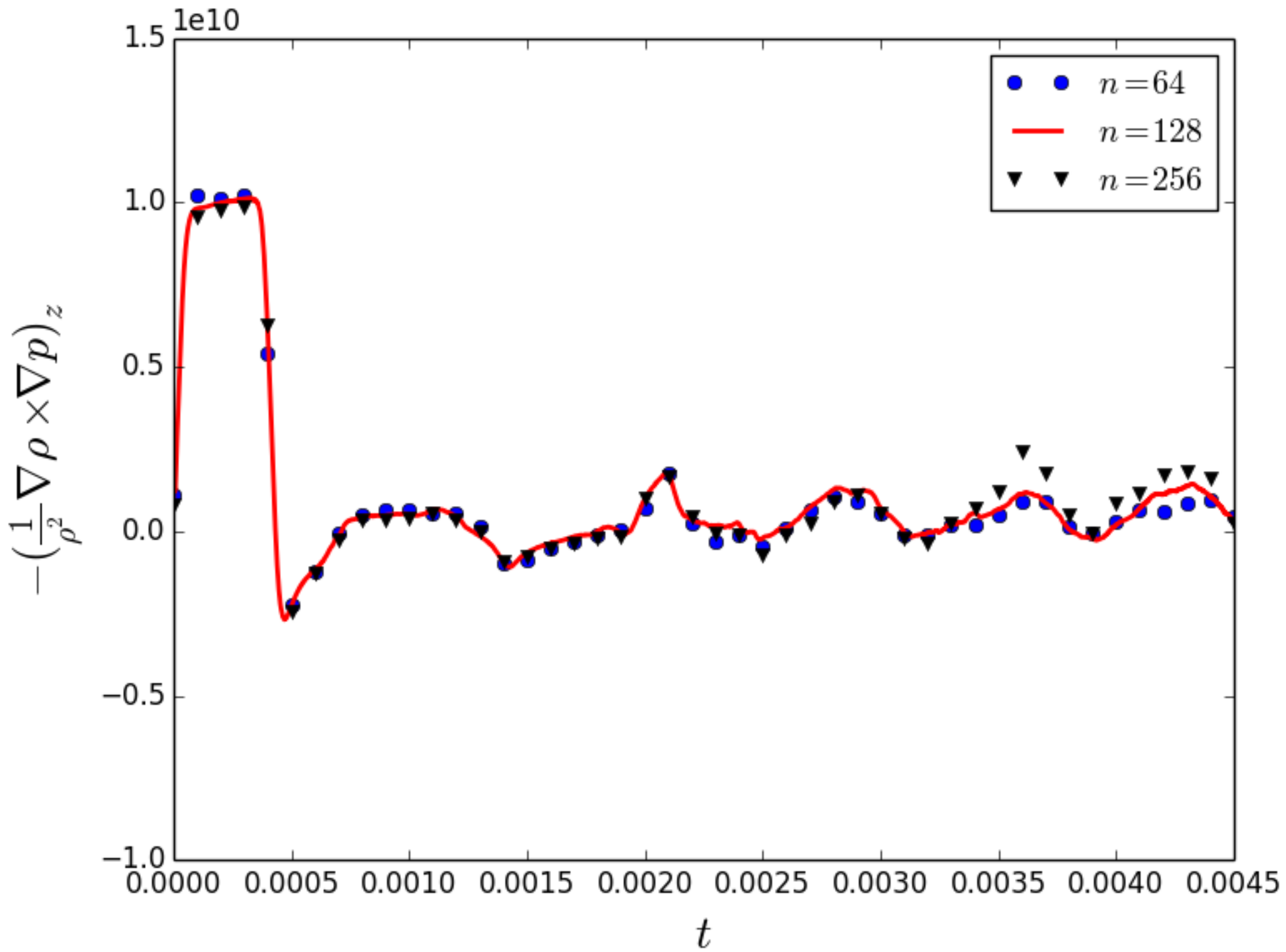
- Turbulent mixing (t = 5.0 ms)



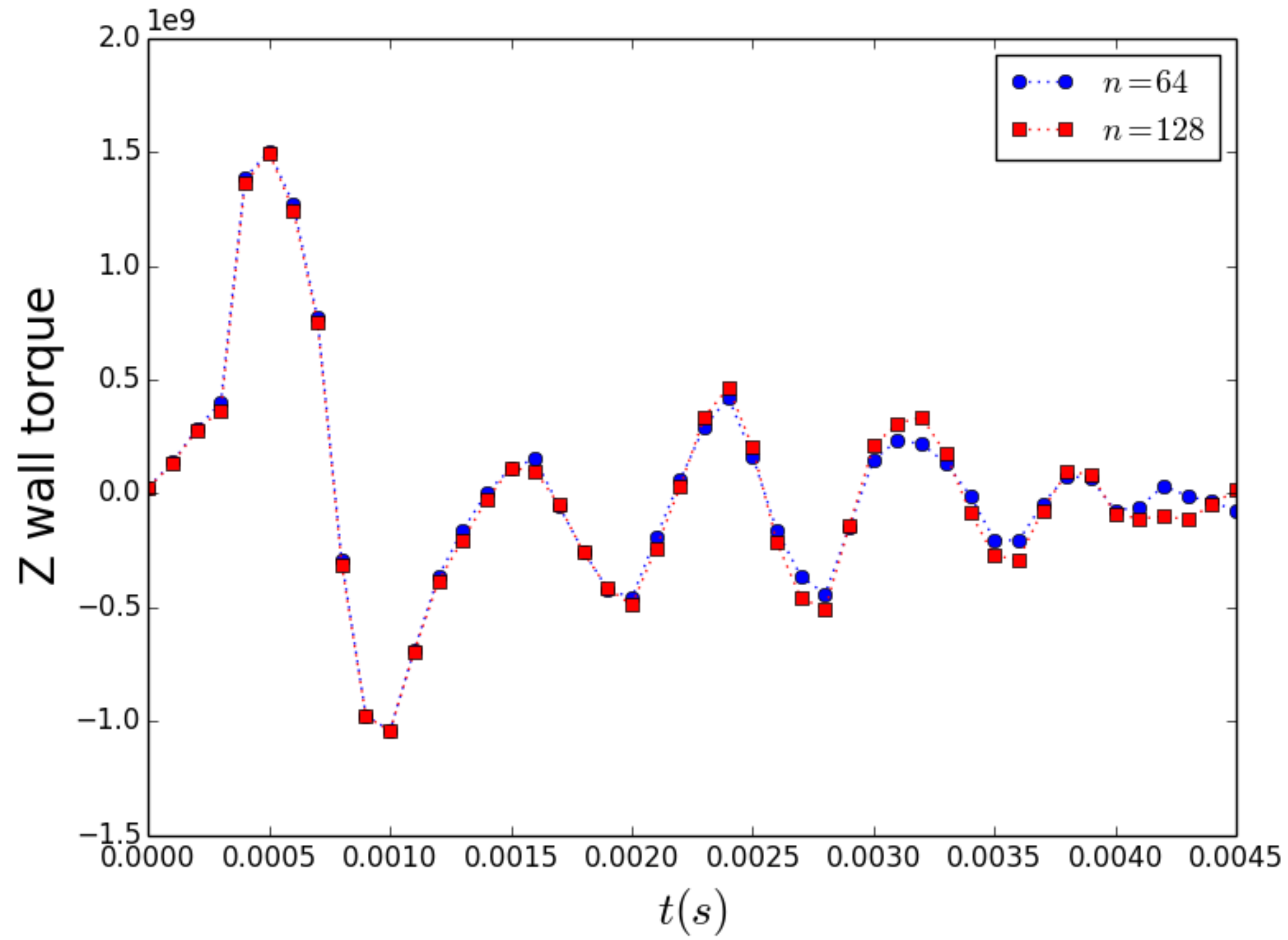
y-z integrated vorticity



Total baroclinic vorticity generation

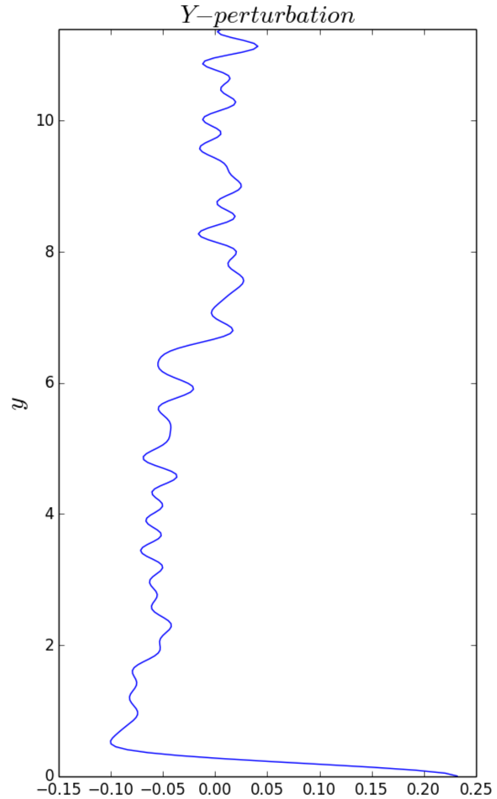


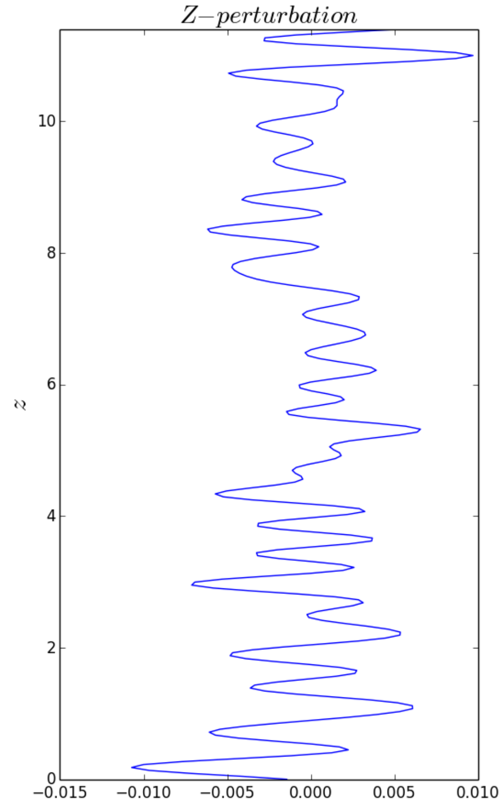
Total wall torque



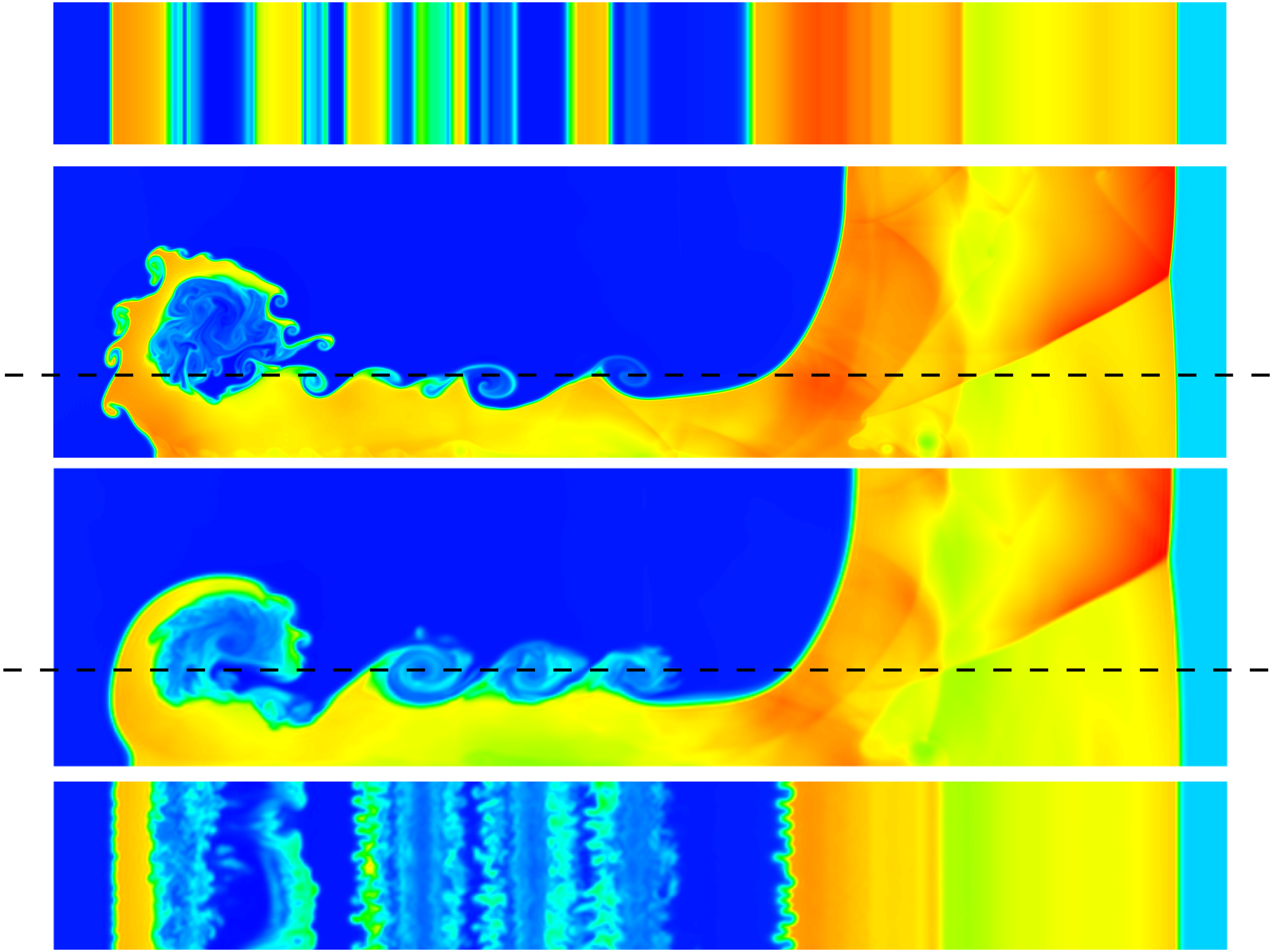
Effect of 3D perturbations

- Quite often, 2D RM simulations are performed since initial conditions are 2D
- Well correlated vortex rolls observed are unrealistic physically
- Want to quantify effects of 3D perturbations on top of the inclined interface
- 3D perturbations informed by more careful profiling of the initial condition data from experiments





- Kelvin-Helmholtz rollers (t = 2.5 ms)



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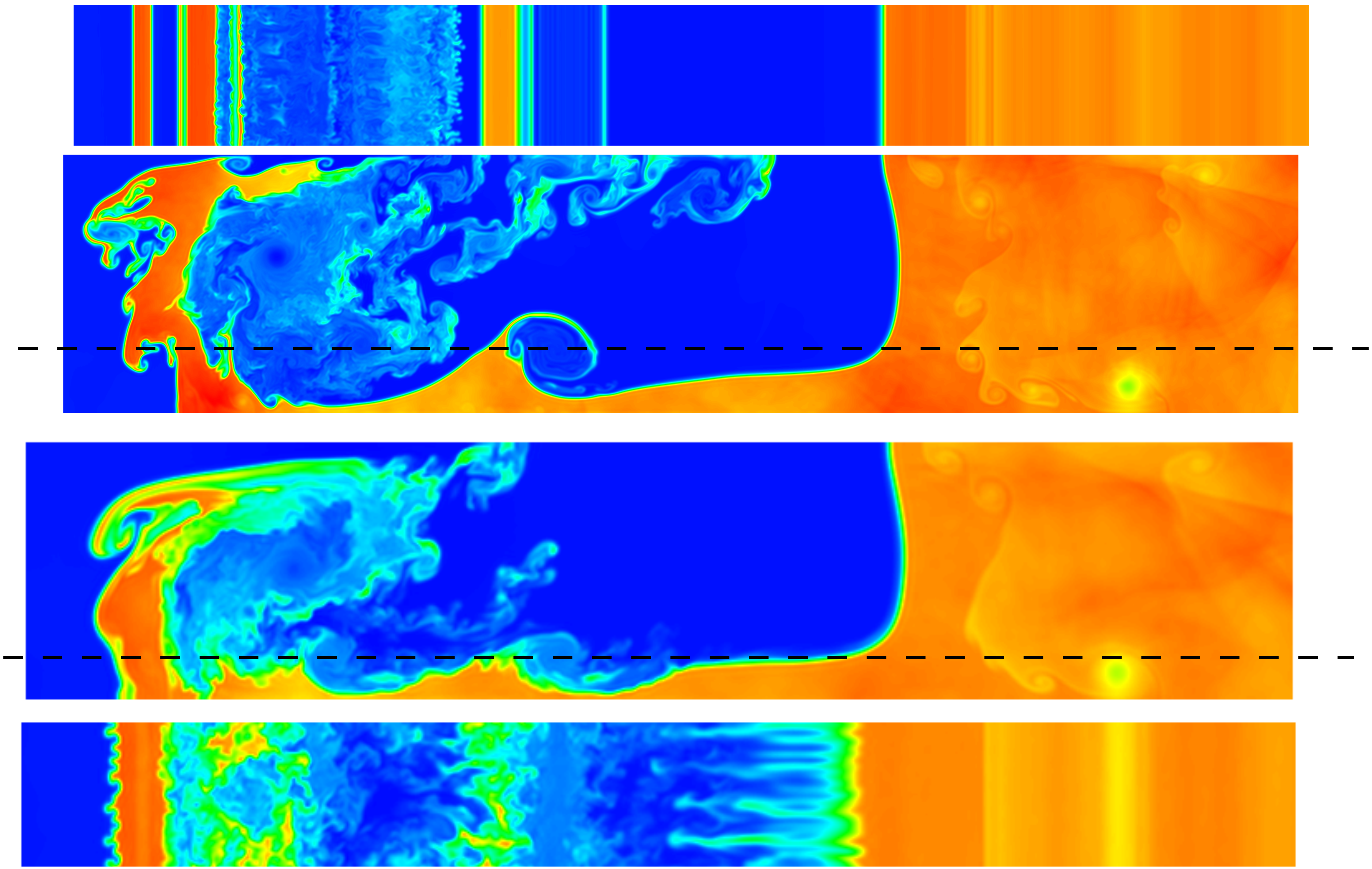

Conclusions and Future Work

- The inclined interface RM problem was simulated for the set of parameter values used in the experiment
- The qualitative physics of the problem are captured well and match what is observed in experiments
- Higher mesh resolution calculations are required to get convergence on higher order statistics
- 3D perturbations play an important role in the vortex breakdown and mixing process
- Next step is to make quantitative comparisons with experiments for validation
- Characterize turbulent mixing by looking at higher order moments

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